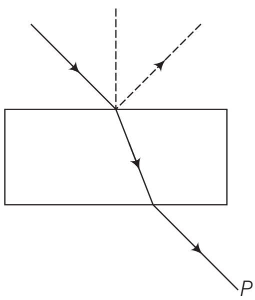
10

Wave Optics

Multiple Choice Questions (MCQs)

Q. 1 Consider a light beam incident from air to a glass slab at Brewster's angle as shown in figure.

A polaroid is placed in the path of the emergent ray at point *P* and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid.



(a) For a particular orientation, there shall be darkness as observed through the polaroid

(b) The intensity of light as seen through the polaroid shall be independent of the rotation

(c) The intensity of light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid

(d) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid

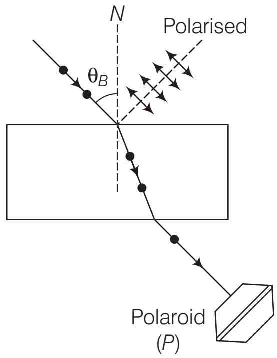
K **Thinking Process**

When the light beam incident at Brewster's angle, the transmitted beam is unpolarised and reflected beam is polarised.

Ans. *(c)* Consider the diagram the light beam incident from air to the glass slab at Brewster's angle (i_p). The incident ray is unpolarised and is represented by dot (.).

The reflected light is plane polarised represented by arrows.

As the emergent ray is unpolarised, hence intensity cannot be zero when passes through polaroid.



Q. 2 Consider sunlight incident on a slit of width 10^4 Å. The image seen through the slit shall

(a) be a fine sharp slit white in colour at the centre
(b) a bright slit white at the centre diffusing to zero intensities at the edges
(c) a bright slit white at the centre diffusing to regions of different colours
(d) only be a diffused slit white in colour

Ans. *(a)* Given, width of the slit $= 10^4$ Å

$$= 10^4 \times 10^{-10} \text{ m} = 10^{-6} \text{ m} = 1 \mu\text{m}$$

Wavelength of (visible) sunlight varies from 4000 Å to 8000 Å.

As the width of slit is comparable to that of wavelength, hence diffraction occurs with maxima at centre. So, at the centre all colours appear *i.e.,* mixing of colours form white patch at the centre.

Q. 3 Consider a ray of light incident from air onto a slab of glass (refractive index n) of width d, at an angle θ. The phase difference between the ray reflected by the top surface of the glass and the bottom surface is

(a) $\frac{4\pi d}{\lambda}\left(1 - \frac{1}{n^2}\sin^2\theta\right)^{1/2} + \pi$

(b) $\frac{4\pi d}{\lambda}\left(1 - \frac{1}{n^2}\sin^2\theta\right)^{1/2}$

(c) $\frac{4\pi d}{\lambda}\left(1 - \frac{1}{n^2}\sin^2\theta\right)^{1/2} + \frac{\pi}{2}$

(d) $\frac{4\pi d}{\lambda}\left(1 - \frac{1}{n^2}\sin^2\theta\right)^{1/2} + 2\pi$

Ans. *(a)* Consider the diagram, the ray (P) is incident at an angle θ and gets reflected in the direction P' and refracted in the direction P''. Due to reflection from the glass medium, there is a phase change of π.

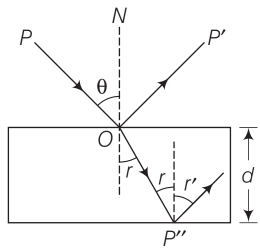
Time taken to travel along OP''

$$\Delta t = \frac{OP''}{v} = \frac{d/\cos r}{c/n} = \frac{nd}{c\cos r}$$

From Snell's law,
$$n = \frac{\sin\theta}{\sin r}$$

$\Rightarrow$
$$\sin r = \frac{\sin\theta}{n}$$

$$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{\sin^2\theta}{n^2}}$$



$$\therefore \qquad \Delta t = \frac{nd}{c\left(1-\frac{\sin^2\theta}{n^2}\right)^{1/2}} = \frac{n^2 d}{c}\left(1-\frac{\sin^2\theta}{n^2}\right)^{-1/2}$$

$$\text{Phase difference} = \Delta\phi = \frac{2\pi}{T}\times\Delta t = \frac{2\pi\, nd}{\lambda}\left(1-\frac{\sin^2\theta}{n^2}\right)^{-1/2}$$

So, net phase difference $= \Delta\phi + \pi$

$$= \frac{4\pi d}{\lambda}\left(1-\frac{1}{n^2}\sin^2\theta\right)^{-1/2} + \pi$$

Q. 4 In a Young's double-slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case,

(a) there shall be alternate interference patterns of red and blue

(b) there shall be an interference pattern for red distinct from that for blue

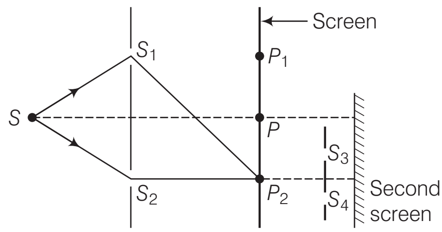
(c) there shall be no interference fringes

(d) there shall be an interference pattern for red mixing with one for blue

Ans. *(c)* For the interference pattern to be formed on the screen, the sources should be coherent and emits lights of same frequency and wavelength.

In a Young's double-slit experiment, when one of the holes is covered by a red filter and another by a blue filter. In this case due to filteration only red and blue lights are present. In YDSE monochromatic light is used for the formation of fringes on the screen. Hence, in this case there shall be no interference fringes.

Q. 5 Figure shows a standard two slit arrangement with slits S_1, S_2, P_1, P_2 are the two minima points on either side of P (figure).



At P_2 on the screen, there is a hole and behind P_2 is a second 2-slit arrangement with slits S_3, S_4 and a second screen behind them.

(a) There would be no interference pattern on the second screen but it would be lighted
(b) The second screen would be totally dark
(c) There would be a single bright point on the second screen
(d) There would be a regular two slit pattern on the second screen

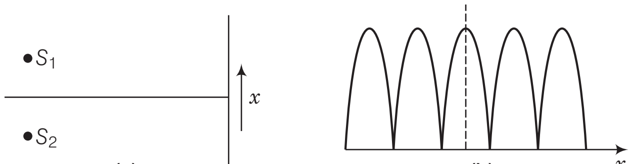
Ans. ***(d)*** According to question, there is a hole at point P_2. From Huygen's principle, wave will propagates from the sources S_1 and S_2. Each point on the screen will acts as secondary sources of wavelets.

Now, there is a hole at point P_2 (minima). The hole will act as a source of fresh light for the slits S_3 and S_4.

Therefore, there will be a regular two slit pattern on the second screen.

Multiple Choice Questions (More Than One Options)

Q. 6 Two sources S_1 and S_2 of intensity I_1 and I_2 are placed in front of a screen [Fig. (a)]. The pattern of intensity distribution seen in the central portion is given by Fig. (b).



(a) (b)

In this case, which of the following statements are true?

(a) S_1 and S_2 have the same intensities
(b) S_1 and S_2 have a constant phase difference
(c) S_1 and S_2 have the same phase
(d) S_1 and S_2 have the same wavelength

Ans. ***(a, b, d)***

Consider the pattern of the intensity shown in the figure

(i) As intensities of all successive minima is zero, hence we can say that two sources S_1 and S_2 are having same intensities.

(ii) As width of the successive maxima (pulses) increases in continuous manner, we can say that the path difference (x) or phase difference varies in continuous manner.

(iii) We are using monochromatic light in YDSE to avoid overlapping and to have very clear pattern on the screen.

Q. 7 Consider sunlight incident on a pinhole of width 10^3 Å. The image of the pinhole seen on a screen shall be

(a) a sharp white ring
(b) different from a geometrical image
(c) a diffused central spot, white in colour
(d) diffused coloured region around a sharp central white spot

Ans. *(b, d)*

Given, width of pinhole $= 10^3$ Å $= 1000$ Å

We know that wavelength of sunlight ranges from 4000 Å to 8000 Å.

Clearly, wavelength $\lambda <$ width of the slit.

Hence, light is diffracted from the hole. Due to diffraction from the slight the image formed on the screen will be different from the geometrical image.

Q. 8 Consider the diffraction pattern for a small pinhole. As the size of the hole is increased

(a) the size decreases
(b) the intensity increases
(c) the size increases
(d) the intensity decreases

Ans. *(a, b)*

(a) When a decreases w increases.
So, size decreases.

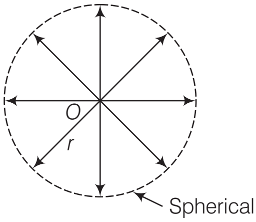
(b) Now, light energy is distributed over a small area and intensity $\propto \frac{1}{\text{area}}$ as area is decreasing so intensity increases.

Q. 9 For light diverging from a point source,

(a) the wavefront is spherical
(b) the intensity decreases in proportion to the distance squared
(c) the wavefront is parabolic
(d) the intensity at the wavefront does not depend on the distance

Ans. *(a, b)*

Consider the diagram in which light diverges from a point source (O).



Due to the point source light propagates in all directions symmetrically and hence, wavefront will be spherical as shown in the diagram.

If power of the source is P, then intensity of the source will be

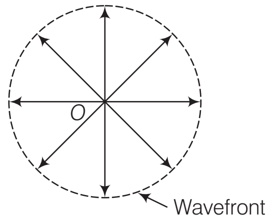
$$I = \frac{P}{4\pi r^2}$$

where, r is radius of the wavefront at any time.

Very Short Answer Type Questions

Q. 10 Is Huygen's principle valid for longitudinal sound waves?

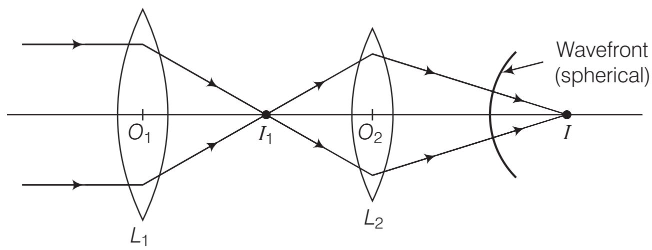
Ans. When we are considering a point source of sound wave. The disturbance due to the source propagates in spherical symmetry that is in all directions. The formation of wavefront is in accordance with Huygen's principle.



So, Huygen's principle is valid for longitudinal sound waves also.

Q. 11 Consider a point at the focal point of a convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of the wavefronts emerging from the final image?

Ans. *Consider the ray diagram shown below*

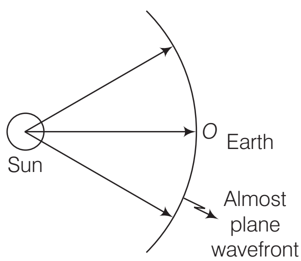


The point image I_1, due to L_1 is at the focal point. Now, due to the converging lense L_2, let final image formed is I which is point image, hence the wavefront for this image will be of spherical symmetry.

Q. 12 What is the shape of the wavefront on earth for sunlight?

Ans. We know that the sun is at very large distance from the earth. Assuming sun as spherical, it can be considered as point source situated at infinity.

Due to the large distance the radius of wavefront can be considered as large (infinity) and hence, wavefront is almost plane.



Q. 13 Why is the diffraction of sound waves more evident in daily experience than that of light wave?

Ans. As we know that the frequencies of sound waves lie between 20 Hz to 20 kHz so that their wavelength ranges between 15 m to 15 mm. The diffraction occur if the wavelength of waves is nearly equal to slit width.

As the wavelength of light waves is 7000×10^{-10} m to 4000×10^{-10} m. The slit width is very near to the wavelength of sound waves as compared to light waves. Thus, the diffraction of sound waves is more evident in daily life than that of light waves.

Q. 14 The human eye has an approximate angular resolution of $\phi = 5.8 \times 10^{-4}$ rad and a typical photoprinter prints a minimum of 300 dpi (dots per inch, 1 inch = 2.54 cm). At what minimal distance z should a printed page be held so that one does not see the individual dots.

Ans. Given, angular resolution of human eye, $\phi = 5.8 \times 10^{-4}$ rad.

and printer prints 300 dots per inch.

The linear distance between two dots is $l = \frac{2.54}{300}$ cm $= 0.84 \times 10^{-2}$ cm.

At a distance of z cm, this subtends an angle, $\phi = \frac{l}{z}$

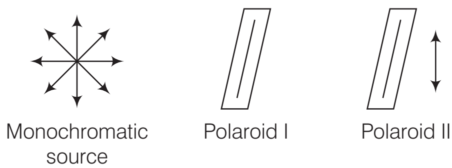
$$\therefore \quad z = \frac{l}{\phi} = \frac{0.84 \times 10^{-2}\text{ cm}}{5.8 \times 10^{-4}} = 14.5\text{ cm}.$$

Q. 15 A polaroid (I) is placed infront of a monochromatic source. Another polariod (II) is placed in front of this polaroid (I) and rotated till no light passes. A third polaroid (III) is now placed in between (I) and (II). In this case, will light emerge from (II). Explain.

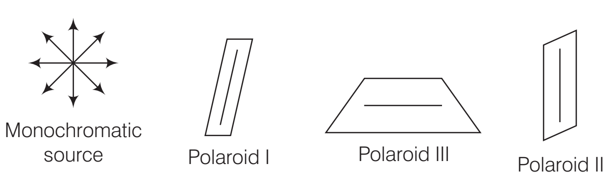
K **Thinking Process**

Natural light e.g., from the sun is unpolariser. This means the electric vector takes all possible direction in the transverse plane, rapidly.

Ans. In the diagram shown, a monochromatic light is placed infront of polaroid (I) as shown below.



As per the given question, monochromatic light emerging from polaroid (I) is plane polarised. When polaroid (II) is placed infront of this polaroid (I), and rotated till no light passes through polaroid (II), then (I) and (II) are set in crossed positions, *i.e.,* pass axes of I and II are at 90°.



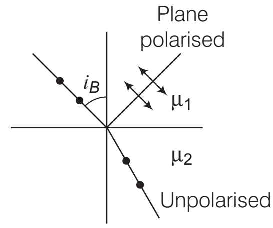
Consider the above diagram where a third polaroid (III) is placed between polaroid (I) and polaroid II.

When a third polaroid (III) is placed in between (I) and (II), no light will emerge from (II), if pass axis of (III) is parallel to pass axis of (I) or (II). In all other cases, light will emerge from (II), as pass axis of (II) will no longer be at 90° to the pass axis of (III).

Short Answer Type Questions

Q. 16 Can reflection result in plane polarised light if the light is incident on the interface from the side with higher refractive index?

Ans. When angle of incidence is equal to Brewster's angle, the transmitted light is unpolarised and reflected light is plane polarised.



Consider the diagram in which unpolarised light is represented by dot and plane polarised light is represented by arrows.

Polarisation by reflection occurs when the angle of incidence is the Brewster's angle

i.e., $$\tan i_B = {}^1\mu_2 = \frac{\mu_2}{\mu_1} \text{ where } \mu_2 < \mu_1$$

when the light rays travels in such a medium, the critical angle is

$$\sin i_c = \frac{\mu_2}{\mu_1}$$

where, $\mu_2 < \mu_1$

As $|\tan i_B| > |\sin i_C|$ for large angles $i_B < i_C$.

Thus, the polarisation by reflection occurs definitely.

Q. 17For the same objective, find the ratio of the least separation between two points to be distinguished by a microscope for light of 5000 Å and electrons accelerated through 100V used as the illuminating substance.

K **Thinking Process**

Resolving power of a microscope is calculated by $\frac{2 \sin\beta}{1.22\,\lambda}$, *with* μ *as refractive index of the medium and* β *is the angle subtented by the objective at the object.*

Ans. We know that

$$\text{Resolving power} = \frac{1}{d} = \frac{2\sin\beta}{1.22\,\lambda} \Rightarrow d_{min} = \frac{1.22\,\lambda}{2\sin\beta}$$

where, λ is the wavelength of light and β is the angle subtended by the objective at the object.

For the light of wavelength 5500 Å,

$$d_{min} = \frac{1.22 \times 5500 \times 10^{-10}}{2\sin\beta} \quad \text{... (i)}$$

For electrons accelerated through 100 V, the de-Broglie wavelength

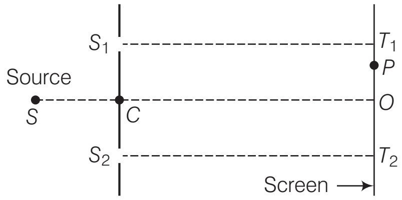
$$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{100}} = 0.12 \times 10^{-9} \text{ m}$$

$$d_{min} = \frac{1.22 \times 0.12 \times 10^{-9}}{2 \sin \beta}$$

Ratio of the least separation

$$\therefore \quad \frac{d'_{min}}{d_{min}} = \frac{0.12 \times 10^{-9}}{5500 \times 10^{-10}} = 0.2 \times 10^{-3}$$

Q. 18 Consider a two slit interference arrangements (figure) such that the distance of the screen from the slits is half the distance between the slits. Obtain the value of D in terms of λ such that the first minima on the screen falls at a distance D from the centre O.



Thinking Process

For nth minima to be formed on the screen path difference between the rays coming from S_1 and S_2 must be $(2n-1)\frac{\lambda}{2}$.

Ans. From the given figure of two slit interference arrangements, we can write

$$T_2P = T_2O + OP = D + x$$

and $$T_1P = T_1O - OP = D - x$$

$$S_1P = \sqrt{(S_1T_1)^2 + (PT_1)^2} = \sqrt{D^2 + (D - x)^2}$$

and $$S_2P = \sqrt{(S_2T_2)^2 + (T_2P)^2} = \sqrt{D^2 + (D + x)^2}$$

The minima will occur when $S_2P - S_1P = (2n - 1)\frac{\lambda}{2}$

i.e., $$[D^2 + (D + x)^2]^{1/2} - [D^2 + (D - x)^2]^{1/2} = \frac{\lambda}{2}$$ [for first minima $n = 1$]

If $$x = D$$

we can write $$[D^2 + 4D^2]^{1/2} - [D^2 + 0]^{1/2} = \frac{\lambda}{2}$$

$$\Rightarrow \quad [5D^2]^{1/2} - [D^2]^{1/2} = \frac{\lambda}{2}$$

$$\Rightarrow \quad \sqrt{5}D - D = \frac{\lambda}{2}$$

$$\Rightarrow \quad D(\sqrt{5} - 1) = \lambda/2 \text{ or } D = \frac{\lambda}{2(\sqrt{5} - 1)}$$

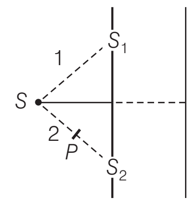
Putting $$\sqrt{5} = 2.236$$

$$\Rightarrow \quad \sqrt{5} - 1 = 2.236 - 1 = 1.236$$

$$D = \frac{\lambda}{2(1.236)} = 0.404\lambda$$

Long Answer Type Questions

Q. 19 Figure shown a two slit arrangement with a source which emits unpolarised light. P is a polariser with axis whose direction is not given. If I_0 is the intensity of the principal maxima when no polariser is present, calculate in the present case, the intensity of the principal maxima as well as of the first minima.



K **Thinking Process**

The resultant amplitude will be the sum of amplitude of either beam in perpendicular and parallel polarisation.

Ans. A = Resultant amplitude

$= A$ parallel $(A_{||}) + A$ perpendicular $(A_\perp)$

$\Rightarrow \qquad A = A_\perp + A_{||}$

Without P $\qquad A = A_\perp + A_{||}$

$$A_1 = A^1{}_\perp + A_\perp^2 = A_\perp^0 \sin(kx - \omega t) + A_\perp^0 \sin(kx - \omega t + \phi)$$

$$A_{||} = A_{||}^{(1)} + A_{||}^{(2)}$$

$$A_{||} = A_{||}^0 [\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

where $A_\perp^0$, $A_{||}^0$ are the amplitudes of either of the beam in perpendicular and parallel polarisations.

$$\therefore \text{Intensity} = \{|A_\perp^0|^2 + |A_{||}{}^0|^2\} [\sin^2(kx - \omega t)(1 + \cos^2\phi + 2\sin\phi) + \sin^2(kx - \omega t)\sin^2\phi]$$

$$= \{|A_\perp^0|^2 + |A_{||}^0|^2\} \left(\frac{1}{2}\right) . 2(1 + \cos\phi)$$

$$= 2|A_\perp^0|^2 (1 + \cos\phi), \text{ since, } |A_\perp^0|_{av} = |A_{||}{}^0|_{av}$$

With P

Assume $A_\perp^2$ is blocked

$$\text{Intensity} = (A_{||}^1 + A_{||}^2)^2 + (A_\perp^1)^2$$

$$= |A_\perp^0|^2 (1 + \cos\phi) + |A_\perp^0|^2 . \frac{1}{2}$$

Given, $I_0 = 4|A_\perp^0|^2$ = Intensity without polariser at principal maxima.

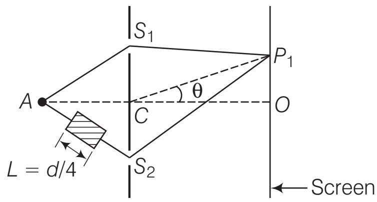
Intensity at principal maxima with polariser

$$= |A_\perp^0|^2 \left(2 + \frac{1}{2}\right) = \frac{5}{8} I_0$$

Intensity at first minima with polariser

$$= |A_\perp^0|^2 (1 - 1) + \frac{|A_\perp^0|^2}{2} = \frac{I_0}{8}.$$

Q. 20



$$AC = CO = D,\ S_1C = S_2C = d << D$$

A small transparent slab containing material of $\mu = 1.5$ is placed along AS_2 (figure). What will be the distance from O of the principal maxima and of the first minima on either side of the principal maxima obtained in the absence of the glass slab?

K **Thinking Process**

Whenever a transparent slab of refractive index μ and thickness t is inserted in the path of the ray the fringes on the screen shifts by $(\mu - 1)t$ towards the slab.

Ans. In case of transparent glass slab of refractive index μ, the path difference will be calculated as $\Delta x = 2d \sin\theta + (\mu - 1) L$.

In case of transparent glass slab of refractive index μ,

the path difference $= 2d \sin\theta + (\mu - 1) L$.

For the principal maxima, (path difference is zero)

i.e., $$2d \sin\theta_0 + (\mu - 1) L = 0$$

or $$\sin\theta_0 = -\frac{L(\mu - 1)}{2d} = \frac{-L(0.5)}{2d} \quad [\because L = d/4]$$

or $$\sin\theta_0 = \frac{-1}{16}$$

$\therefore$ $$OP = D\tan\theta_0 \approx D\sin\theta_0 = \frac{-D}{16}$$

For the first minima, the path difference is $\pm\frac{\lambda}{2}$

$\therefore$ $$2d \sin\theta_1 + 0.5L = \pm\frac{\lambda}{2}$$

or $$\sin\theta_1 = \frac{\pm\lambda/2 - 0.5L}{2d} = \frac{\pm\lambda/2 - d/8}{2d}$$

$$= \frac{\pm\lambda/2 - \lambda/8}{2\lambda} = \pm\frac{1}{4} - \frac{1}{16}$$

[$\because$ The diffraction occurs if the wavelength of waves is nearly equal to the side width (d)]

On the positive side $\sin\theta'^{+}_1 = +\frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

On the negative side $\sin\theta''^{-}_1 = -\frac{1}{4} - \frac{1}{16} = -\frac{5}{16}$

The first principal maxima on the positive side is at distance

$$D\tan\theta'^{+}_1 = D\frac{\sin\theta'^{+}_1}{\sqrt{1 - \sin^2\theta'_1}} = D\frac{3}{\sqrt{16^2 - 3^2}} = \frac{3D}{\sqrt{247}} \text{ above point } O$$

The first principal minima on the negative side is at distance

$$D\tan\theta''_1 = \frac{5D}{\sqrt{16^2 - 5^2}} = \frac{5D}{\sqrt{231}} \text{ below point } O.$$

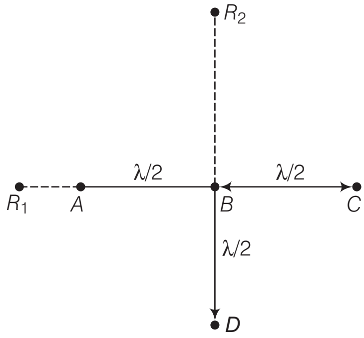
Q. 21 Four identical monochromatic sources *A, B, C, D* as shown in the (figure) produce waves of the same wavelength λ and are coherent. Two receiver R_1 and R_2 are at great but equal distances from *B*.

(i) Which of the two receivers picks up the larger signal?

(ii) Which of the two receivers picks up the larger signal when *B* is turned off?

(iii) Which of the two receivers picks up the larger signal when *D* is turned off?

(iv) Which of the two receivers can distinguish which of the sources *B* or *D* has been turned off?



$$R_1B = d = R_2\ B$$
$$AB = BC = BD = \lambda/2$$

K Thinking Process

The resultant disturbance at a point will be calculated by some of disturbances due to individual sources.

Ans. Consider the disturbances at the receiver R_1 which is at a distance *d* from *B*.

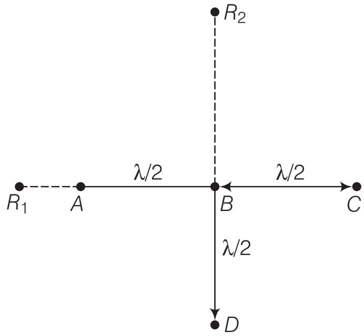
Let the wave at R_1 because of *A* be $Y_A = a\cos\omega t$. The path difference of the signal from *A* with that from *B* is $\lambda/2$ and hence, the phase difference is π.

Thus, the wave at R_1 because of *B* is

$$y_B = a\cos(\omega t - \pi) = -a\cos\omega t.$$

The path difference of the signal from *C* with that from *A* is λ and hence the phase difference is 2π.

Thus, the wave at R_1 because of *C* is $Y_c = a\cos(\omega t - 2\pi) = a\cos\omega t$



The path difference between the signal from D with that of A is

$$\sqrt{d^2 + \left(\frac{\lambda}{2}\right)^2} - (d - \lambda/2) = d\left(1 + \frac{\lambda}{4d^2}\right)^{1/2} - d + \frac{\lambda}{2}$$

$$= d\left(1 + \frac{\lambda^2}{8d^2}\right)^{1/2} - d + \frac{\lambda}{2} \approx \frac{\lambda}{2} \qquad (\because d >> \lambda)$$

Therefore, phase difference is π.

$\therefore \qquad Y_D = a\cos(\omega t - \pi) = -a\cos\omega t$

Thus, the signal picked up at R_1 from all the four sources is $Y_{R_1} = y_A + y_B + y_C + y_D$

$$= a\cos\omega t - a\cos\omega t + a\cos\omega t - a\cos\omega t = 0$$

(i) Let the signal picked up at R_2 from B be $y_B = a_1 \cos\omega t$.

The path difference between signal at D and that at B is $\lambda/2$.

$\therefore \qquad y_D = -a_1\cos\omega t$

The path difference between signal at A and that at B is

$$\sqrt{(d)^2 + \left(\frac{\lambda}{2}\right)^2} - d = d\left(1 + \frac{\lambda^2}{4d^2}\right)^{1/2} - d \simeq \frac{1}{8}\frac{\lambda^2}{d^2}$$

As $d \gg \lambda$, therefore this path difference $\to 0$

and $\qquad$ phase difference $= \dfrac{2\pi}{\lambda}\left(\dfrac{1}{8}\dfrac{\lambda^2}{d^2}\right) \to 0$

Hence, $\qquad y_A = a_1\cos(\omega t - \phi)$

Similarly, $\qquad y_C = a_1\cos(\omega t - \phi)$

$\therefore$ Signal picked up by R_2 is

$$y_A + y_B + y_C + y_D = y = 2a_1\cos(\omega t - \phi)$$

$\therefore \qquad |y|^2 = 4a_1^2\cos^2(\omega t - \phi)$

$\therefore \qquad \langle I \rangle = 2a_1^2$

Thus, R_1 picks up the larger signal.

(ii) If B is switched off,

R_1 picks up $\qquad y = a\cos\omega t$

$\therefore \qquad \langle I_{R_1} \rangle = \dfrac{1}{2}a^2$

R_2 picks up $\qquad y = a\cos\omega t$

$\therefore \qquad \langle I_{R_2} \rangle = a^2 \langle\cos^2\omega t\rangle = \dfrac{a^2}{2}$

(iii) Thus, R_1 and R_2 pick up the same signal.

If D is switched off.

R_1 picks up $y = a\cos\omega t$

$\therefore \qquad \langle I_{R_1} \rangle = \dfrac{1}{2}a^2$

R_2 picks up $\qquad y = 3a\cos\omega t$

$\therefore \qquad \langle I_{R_2} \rangle = 9a^2 \langle\cos^2\omega t\rangle = \dfrac{9a^2}{2}$

Thus, R_2 picks up larger signal compared to R_1.

(iv) Thus, a signal at R_1 indicates B has been switched off and an enhanced signal at R_2 indicates D has been switched off.

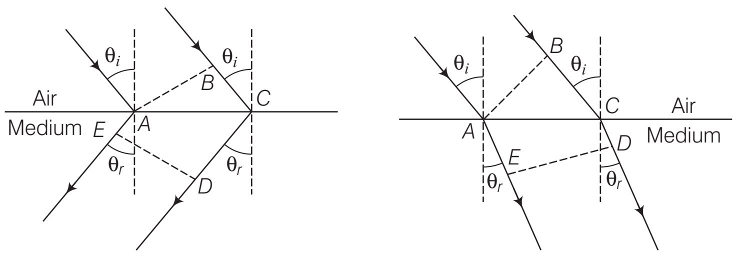
Q. 22 The optical properties of a medium are governed by the relative permittivity (ε_r) and relative permeability (μ_r). The refractive index is defined as $\sqrt{\mu_r \varepsilon_r} = n$. For ordinary material, $\varepsilon_r > 0$ and $\mu_r > 0$ and the positive sign is taken for the square root.

In 1964, a Russian scientist V. Veselago postulated the existence of material with $\varepsilon_r < 0$ and $\mu_r < 0$. Since, then such metamaterials have been produced in the laboratories and their optical properties studied. For such materials $n = -\sqrt{\mu_r \varepsilon_r}$. As light enters a medium of such refractive index the phases travel away from the direction of propagation.

(i) According to the description above show that if rays of light enter such a medium from air (refractive index = 1) at an angle θ in 2nd quadrant, then the refracted beam is in the 3rd quadrant.

(ii) Prove that Snell's law holds for such a medium.

Ans. Let us assume that the given postulate is true, then two parallel rays would proceed as shown in the figure below



(i) Let AB represent the incident wavefront and DE represent the refracted wavefront. All points on a wavefront must be in same phase and in turn, must have the same optical path length.

Thus $$-\sqrt{\varepsilon_r \mu_r}\, AE = BC - \sqrt{\varepsilon_r \mu_r}\, CD$$

or $$BC = \sqrt{\varepsilon_r \mu_r}\,(CD - AE)$$

$$BC > 0,\ CD > AE$$

As showing that the postulate is reasonable. If however, the light proceeded in the sense it does for ordinary material (*viz*. in the fourth quadrant, Fig. 2)

Then, $$-\sqrt{\varepsilon_r \mu_r}\, AE = BC - \sqrt{\varepsilon_r \mu_r}\, CD$$

or $$BC = \sqrt{\varepsilon_r \mu_r}\,(CD - AE)$$

If $BC > 0$, then $CD > AE$

which is obvious from Fig (i).

Hence, the postulate reasonable.

However, if the light proceeded in the sense it does for ordinary material, (going from 2nd quadrant to 4th quadrant) as shown in Fig. (i)., then proceeding as above,

$$-\sqrt{\varepsilon_r \mu_r}\, AE = BC - \sqrt{\varepsilon_r \mu_r}\, CD$$

or $$BC = \sqrt{\varepsilon_r \mu_r}\,(CD - AE)$$

As $AE > CD$, therefore $BC < 0$ which is not possible. Hence, the given postulate is correct.

(ii) From Fig. (i)

and $\quad BC = AC \sin \theta_i$

$$CD - AE = AC \sin \theta_r$$

As $\quad BC = \sqrt{\mu_r \varepsilon_r} \quad [CD - AE = BC]$

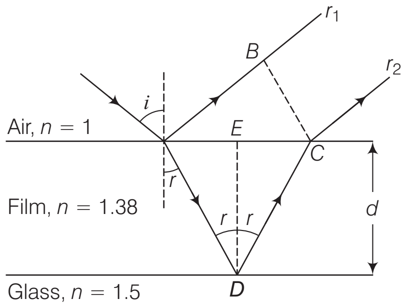
$\therefore \quad AC \sin \theta_i = \sqrt{\varepsilon_r \mu_r}\; AC \sin \theta_r$

or $\quad \dfrac{\sin \theta_i}{\sin \theta_r} = \sqrt{\varepsilon_r\, \mu_r} = n$

Which proves Snell's law.

Q. 23 To ensure almost 100% transmittivity, photographic lenses are often coated with a thin layer of dielectric material. The refractive index of this material is intermediated between that of air and glass (which makes the optical element of the lens). A typically used dielectric film is MgF_2 ($n = 1.38$). What should the thickness of the film be so that at the centre of the visible spectrum (5500 Å) there is maximum transmission.

Ans. In this figure, we have shown a dielectric film of thickness d deposited on a glass lens.



Refractive index of film = 1.38 and refractive index of glass = 1.5.

Given, $\quad \lambda = 5500$ Å .

Consider a ray incident at an angle i. A part of this ray is reflected from the air-film interface and a part refracted inside.

This is partly reflected at the film-glass interface and a part transmitted. A part of the reflected ray is reflected at the film-air interface and a part transmitted as r_2 parallel to r_1. Of course successive reflections and transmissions will keep on decreasing the amplitude of the wave.

Hence, rays r_1 and r_2 shall dominate the behaviour. If incident light is to be transmitted through the lens, r_1 and r_2 should interfere destructively. Both the reflections at A and D are from lower to higher refractive index and hence, there is no phase change on reflection. The optical path difference between r_2 and r_1 is

$$n(AD + CD) - AB$$

If d is the thickness of the film, then

$$AD = CD = \frac{d}{\cos r}$$

$$AB = AC \sin i$$

$$\frac{AC}{2} = d \tan r$$

$\therefore \quad AC = 2d \tan r$

Hence, $AB = 2d \tan r \sin i$.

Thus, the optical path difference $= \dfrac{2nd}{\cos r} - 2d \tan r \sin i$

$$= 2 \cdot \frac{\sin i \, d}{\sin r \cos r} - 2d \frac{\sin r}{\cos r} \sin i$$

$$= 2d \sin \left[\frac{1 - \sin^2 r}{\sin r \cos r} \right]$$

$$= 2nd \cos r$$

For these waves to interfere destructively path difference $= \dfrac{\lambda}{2}$.

$$\Rightarrow \quad 2nd \cos r = \frac{\lambda}{2}$$

$$\Rightarrow \quad nd \cos r = \frac{\lambda}{4} \quad \text{... (i)}$$

For photographic lenses, the sources are normally in vertical plane

$$\therefore \quad i = r = 0^\circ$$

From Eq. (i), $\quad nd \cos 0^\circ = \dfrac{\lambda}{4}$

$$\Rightarrow \quad d = \frac{\lambda}{4n}$$

$$= \frac{5500 \text{ Å}}{4 \times 1.38} \approx 1000 \text{ Å}$$